Numerals are doubly bounded: evidence from exclusives and polarity

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What is the status of this inference?

(1) Three cats won awards.

→ No more than three cats won awards.

Scalar implicature?

(2)

a. $\exists x[\textbf{cats}(x) \land \textbf{won}(\textit{awards})(x) \land \#(x) = 3]$

b. $\neg\exists y[\textbf{cats}(y) \land \textbf{won}(\textit{awards})(y) \land \#(y) > 3]$

(Horn 1972, Spector 2013, Bylinina & Nouwen 2018)

Maximization over degrees?

(3) $max[n|\exists x[\mathbf{cats}(x) \wedge \mathbf{won}(awards)(x) \wedge \#(x) = n]] = 3$

(Kennedy 2015, Buccola & Spector 2016)

This talk

- English exclusives *just*, *merely*, and *mere* can license negative polarity items, but only when they compose with degree terms.

- To account for this data, we need a degree-maximizing theory of numerals that computes upper bounds in the semantics.

Data

(4)

a. **Just six** American skiers have ever won two medals in a single Olympics.
b. **Just two** were ever made.
c. The Librarian was one of **just three** Lifeworkers ever honored with that rank.
d. From Friday to Wednesday, over the course of twelve screenings, **merely 95** seats ever sat empty.
e. **A mere 750** were ever subsequently convicted of any offense.

Data

- Coppock & Beaver (2013) analyze *a mere n* as a generalized quantifier whose at-issue component is downward entailing.

- However, not just any quantifier will do: it has to be a degree term.

Data

(5)

a. Just three of my cats have ever won any awards.
b. Just a few of my cats have ever won any awards.
c. Just a minority of my cats have ever won any awards.
d. Just a fraction of my cats have ever won any awards.
e. #Just some of my cats have ever won any awards.
f. #Just a cat won any awards.
g. #Just Russian blues won any awards.

Data

Just and *merely* don’t license NPIs as VP-modifiers either (unlike *only*).

(6)

a. My cat Gertrude only ever eats caviar.
b. #My cat Gertrude just ever eats caviar.
c. #My cat Gertrude merely ever eats caviar.

Data

A final puzzle: numerals need to be focused, and they need to compose directly.

(7)

a. I brought just $[\text{three}]_F$ pencils to any of the exams.
b. #I brought just three $[\text{pencils}]_F$ to any of the exams.
c. #I just brought three pencils to any of the exams.

Roadmap

This data provides evidence for two theoretically significant conclusions: one about how exclusives vary, and one about how numeral upper bounds are derived.

- *Just*, *merely*, and *mere* do not order alternatives by entailment (unlike *only*).
- Numerals impose their own upper bounds over degrees.
- The lower bounds presupposed by exclusives and the upper bounds asserted by numerals conspire to create an NPI environment.

Exclusives

(8) Lexical entry schema for exclusives (Coppock & Beaver 2014)

a. $\textsc{min}(p) = \lambda w. \exists q \in_{\text{CQ}} [q(w) \wedge q \geq p]$
b. $\textsc{max}(p) = \lambda w. \forall q \in_{\text{CQ}} [q(w) \rightarrow p \geq q]$
c. $[\![only]\!] = \lambda p \lambda w\text{: } \textsc{min}(p)(w).\textsc{max}(p)(w)$

Intended to unify **complement exclusion** and **scalar** readings.

Exclusives

Variation in the ≥ relation results in different readings.

(9) Gertrude **only** eats caviar.

→ Gertrude eats nothing **other** than caviar. // *entailment*(≥)

(10) Frederick is **just** a kitten.

→ Frederick is nothing **higher** than a kitten. // *rank*(≥)

How absolute are scalar restrictions?

- Horn (2000): *only* orders alternatives by entailment, *just* by rank.
- Coppock & Beaver (2014): exclusives have “soft preferences” for different scales.

Fagen (2022): by restricting *just*/*merely* to rank-order scales, we can derive the NPI pattern.

NPIs

Chierchia (2013): NPIs are existential quantifiers associated with maximally wide domains, that trigger exhaustification over domain alternatives.

(11) [[ever]] = $\lambda e.\, \exists\, i \subset_{\mathbf{ever'}} [\tau(e) = i]$

(12)

a. *exh*[Gertrude doesn't **ever** eat kibble.]

b. # *exh*[Gertrude ever **eats** kibble.]

NPIs

(11) $[[exh]] = \lambda p \lambda w.p(w) \wedge \forall q \in \text{ALT}(p)[{}^a q(w) \rightarrow {}^a p \geq {}^a q]$

- Exhaustification is **scalar**: sensitive to the same orderings exclusives are.
- This allows a straightforward treatment of rank-order scales.

What are the alternatives?

- Exclusive's focus alternatives (F-ALT)
- NPI's domain alternatives (D-ALT)
- The propositional F-ALTs will also include NPIs, so we need to include the D-ALTs for each F-ALT too.

$\text{ALT}(p) = \text{F-ALT}(p) \cup \text{D-ALT}(p) \cup \{\text{D-ALT}(q) \mid q \in \text{F-ALT}(p)\}$

#just ever

MAX does not reverse strength: if $q \geq p$, then $\textsc{max}(q) \geq \textsc{max}(p)$. This means the narrower D-ALTs are still ranked higher than the prejacent.

(13) #[[*exh*(Gertrude just ever eats caviar)]]

$= (\exists i \subset_{\textbf{ever}} [\tau(\textbf{eat}(\textbf{k})(\textbf{g})) = i])(\lambda p \lambda w : \textsc{min}(p)(w).\ \text{MAX}(p)(w) \wedge$

$\forall q \in \textsc{alt}(\textsc{max}(p))[q(\text{w}) \rightarrow \textsc{max}(p) \geq q])$

#just ever

(14)

a. ALT(*p*) = {<*caviar, ever*>, ~~<*kibble, ever*>~~, ~~<*chocolate, ever*>~~, <*caviar, sometimes*>, <*kibble, sometimes*>, <*chocolate, sometimes*>, <*caviar, often*>, <*kibble, often*>, <*chocolate, often*>...}

b. ALT(MAX(*p*)) = {MAX(<*caviar, ever*>), ~~MAX(<*caviar, sometimes*>)~~, ~~MAX(<*caviar, often*>~~)...}

✗ contradiction!

Proposal

- Without an independent NPI trigger, NPIs cannot appear in the scope of rank-order MIN/MAX.
- Contra Coppock & Beaver, then, NPIs with *just n* must be outside the MIN/MAX environment.
- Scope of *just* is confined to the numeral, but NPIs are still in the scope of the degree-maximizing function.

(15)

a. Just three cats won any awards.
b. #Just some cats won any awards.

Proposal

Numerals as existential quantifiers:

(15) ONLY($\exists x$[**cats**(x) $\wedge$ **won**(*awards*)(x) $\wedge$ #(x) = 3])

✗ still in the MIN/MAX environment.

Proposal

Numerals as degree quantifiers (Kennedy 2015):

(16)

a. $[[\textit{three}]] = \lambda D_{\langle d,p\rangle}.max[n|D(n)] = 3$
b. $\text{ONLY}(max[n|\exists x[\textbf{cats}(x) \wedge \textbf{won}(awards)(x) \wedge \#(x) = n]] = 3)$

✗ still in the MIN/MAX environment.

Proposal

To get the right LF, we need to decompose numerals and maximality.

Buccola & Spector (2016):

(17)

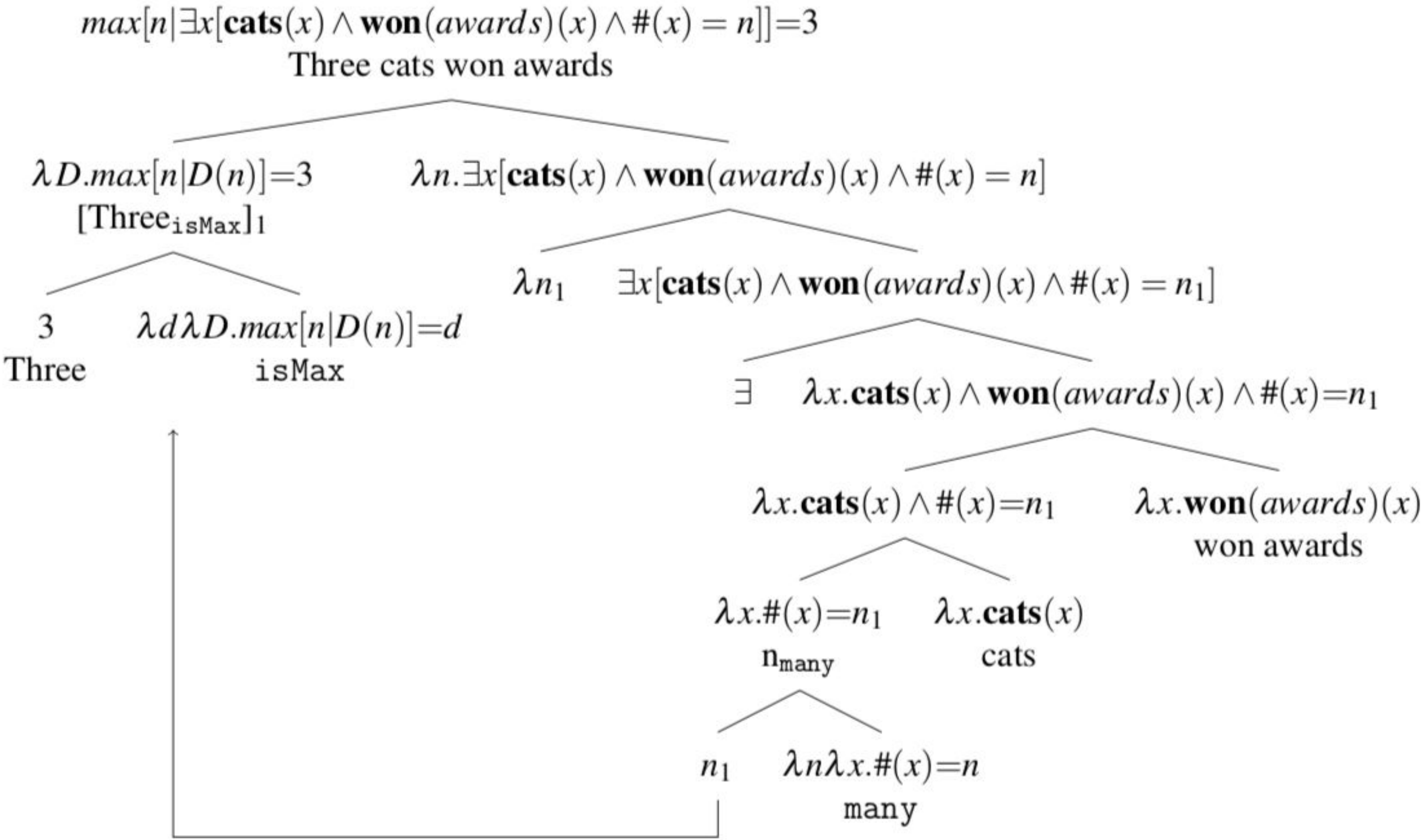
a. $[[\textit{three}]] = 3$

b. $[[\textit{many}]] = \lambda d_d \lambda x_e.\#(x) = d$

c. $[[\textit{isMax}]] = \lambda d_d \lambda D_{\langle d,p \rangle}.max[n|D(n)] = d$

d. $max(D_{\langle d,p \rangle}) =$ if $\exists n[D(n)]$: $\iota n.D(n) \wedge \forall m[D(m) \rightarrow m \leq n]$

else 0

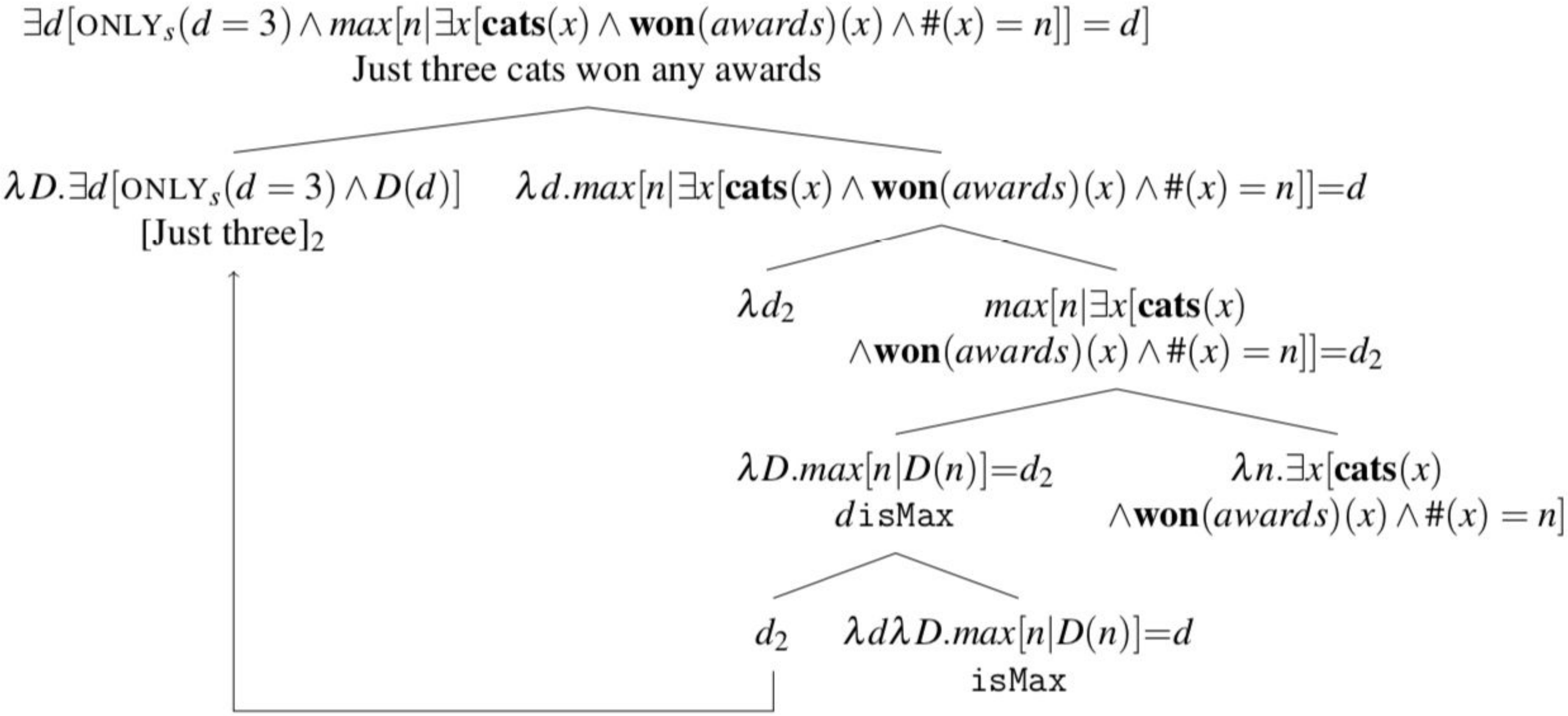


$max[n|\exists x[\textbf{cats}(x) \wedge \textbf{won}(awards)(x) \wedge \#(x) = n]]{=}3$
Three cats won awards
$\lambda D.max[n|D(n)]{=}3$
$[\text{Three}_{\texttt{isMax}}]_1$
$\lambda n.\exists x[\textbf{cats}(x) \wedge \textbf{won}(awards)(x) \wedge \#(x) = n]$
3
Three
$\lambda d \lambda D.max[n|D(n)]{=}d$
isMax
λn_1
$\exists x[\textbf{cats}(x) \wedge \textbf{won}(awards)(x) \wedge \#(x) = n_1]$
$\exists$
$\lambda x.\textbf{cats}(x) \wedge \textbf{won}(awards)(x) \wedge \#(x){=}n_1$
$\lambda x.\textbf{cats}(x) \wedge \#(x){=}n_1$
$\lambda x.\textbf{won}(awards)(x)$
won awards
$\lambda x.\#(x){=}n_1$
$\text{n}_{\texttt{many}}$
$\lambda x.\textbf{cats}(x)$
cats
n_1
$\lambda n \lambda x.\#(x){=}n$
many

Proposal

(18)

a. $[[\text{just}_{\text{D-ONLY}}]] = \lambda D_{\langle d,p\rangle} \lambda n_{\text{d}}.\text{ONLY}(D(n))$
b. $\text{LIFT}(3) = \lambda D.D(3)$
c. $\text{BE}(\text{LIFT}(3) = \lambda n.n = 3$
d. $[[\textit{just}]]((\text{BE}(\text{LIFT}(3))) = \lambda n.\text{ONLY}(n = 3)$
e. $\text{A}(\lambda n.\text{ONLY}(n = 3)) = \lambda D.\exists d[\text{ONLY}(d = 3) \wedge D(d)]$



∃d[ONLY_s(d = 3) ∧ max[n|∃x[cats(x) ∧ won(awards)(x) ∧ #(x) = n]] = d]
Just three cats won any awards
λD.∃d[ONLY_s(d = 3) ∧ D(d)]
[Just three]_2
λd.max[n|∃x[cats(x) ∧ won(awards)(x) ∧ #(x) = n]]=d
λd_2
max[n|∃x[cats(x)
∧won(awards)(x) ∧ #(x) = n]]=d_2
λD.max[n|D(n)]=d_2
disMax
λn.∃x[cats(x)
∧won(awards)(x) ∧ #(x) = n]
d_2
λdλD.max[n|D(n)]=d
isMax

Proposal

- Same truth-conditions as before, but the NPI has been rescued from the MIN/MAX environment.
- Ordering on the D-ALTs can default to entailment now, as desired.

(19) $exh[\exists d[\text{MAX}(d = 3) \land max[n|\exists x[\mathbf{cats}(x) \land \mathbf{won}(awards)(x) \land \#(x) = n]] = d]] = p \land \forall q \in \text{D-ALT}(p)[(p \nrightarrow q) \rightarrow \neg q]$

✓ NPIs ok!

Proposal

- Exclusives are necessary to render the at-issue component downward entailing, rather than non-monotonic.
- Numeral maximality is necessary to bound NPIs outside the MIN/MAX environment.

(20) $\exists d[\text{ONLY}(d = 3) \wedge \exists x[\mathbf{cats}(x) \wedge \mathbf{won}(\mathit{awards})(x) \wedge \#(x) = d]]$

✗ Without maximality, at-issue component of (20) is trivial.

Proposal

(21)

a. I brought just [three]$_F$ pencils to any of the exams.
b. #I brought just three [pencils]$_F$ to any of the exams.
c. #I just brought three pencils to any of the exams.

- Exclusives and numerals need to compose directly so they can scope together.
- Focus must be on the numeral because it's the only expression in the MIN/MAX environment.

Conclusions

- English exclusives *just*, *merely*, and *mere* can license negative polarity items, but only when they compose with degree terms.

- To explain this data in a framework that predicts polarity contrasts between exclusives more generally, we need a numeral semantics that maximizes over degrees, without which the unusual NPI-licensing properties of these exclusives would go unexplained.

Conclusions

- We need stricter restrictions on how exclusives order alternatives: *just*, *merely*, and *mere* are restricted to rank-order scales.

- Numerals denote singular degree terms, but can be shifted to upper-bounding degree quantifiers.

Thanks!

Buccola, Brian, and Benjamin Spector. 2016. Modified numerals and maximality. *Linguistics and Philosophy* 39:151–199.

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Spector, Benjamin. 2013. Bare numerals and scalar implicatures. *Language and Linguistics Compass* 7:273–294.

Bonus: Bylinina & Nouwen (2018)

Why doesn't *zero* license NPIs?

(22)

a. No cats won any awards.
b. #Zero cats won any awards.

Bonus: Bylinina & Nouwen (2018)

(23) $\exists x[\mathbf{cats}(x) \land \mathbf{won}(\mathit{awards})(x) \land \#(x) = 0]$

- This is a contradiction given traditional semantics of plural count nouns as a join semi-lattice.
- BN propose to augment plural noun denotations with a bottom element $\bot$.

Bonus: Bylinina & Nouwen (2018)

- With $\bot$, *zero* sentences are tautologies, but become contingent via exhaustification.
- NPIs are bad: (24) is not downward entailing prior to application of *exh*.

(24) [[*exh*[Zero cats won awards]]]

$= \exists x[\mathbf{cats}(x) \land \mathbf{won}(\mathit{awards})(x) \land \#(x) = 0] \land \neg\exists y[\mathbf{cats}(y) \land$

$\mathbf{won}(\mathit{awards})(y) \land \#(y) > 0]$

Bonus: Bylinina & Nouwen (2018)

(25) $max[n|\exists x[\mathbf{cats}(x) \wedge \mathbf{won}(awards)(x) \wedge \#(x) = n]] = 0$

- The doubly bounded (25) derives the right truth-conditions, but wrongly predicts *zero* to license NPIs.
- BN argue that *zero*'s polarity profile is evidence for a lower-bounded existential numeral semantics that does not maximize over degrees.

Bonus: Bylinina & Nouwen (2018)

- Given upper-bounded numeral semantics, we expect *zero* to license NPIs because it's the lowest value in the domain.
- But as the data I've considered in this paper shows, positive numerals **do** license NPIs when it's presupposed that they denote the lowest value in the domain.
- Data with *zero* and positive numerals points in different directions!